Determinants of the Optimal and Symmetricity of the Volatility Model for Some Selected Nigerian Stocks

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Abstract
Business data always exhibits certain degree of asymmetry and some heteroscedasticity which affect the models and model choice. This asymmetry can be improved to improve the forecasts performance of the model. This work considered cases whereby the errors in the model are normally distributed or normally violate assumption, to improve the efficiency in parameter estimation and forecast as well. The results show that to determine whether volatility require the normal inverse Gaussian distribution, non-normality, and asymmetry which very important in modelling financial returns. Common model selection criteria, the Akaike Information Criterion (AIC) and the Swartz-Bayesian Information Criterion (BIC), were used to determine which recommended model best fits the price and return series to choose. The results show that the best marginal values of the residuals of the GARCH model when estimating Dancem, GTCO, Vitafoam, Nestle, and Fidson were normal inverse Gaussian distributions, respectively.

***Key words**: Volatility, Heteroskedastics, Leverage effect, Log-returns, Nigerian Stocks.*

1. Introduction:

Volatility is important to the functioning of financial markets and is considered a barometer of uncertainty when investing in financial assets. Policy makers, financial industry regulators, and investors are concerned about volatility. A fact about financial volatility is that bad news (negative shocks) tends to have a greater impact on volatility than good news (positive shocks). Black (1976) attributed this effect to the fact that bad news tends to lower stock prices. This increases leverage. Based on this, the asymmetric news effect is commonly referred to as the leverage effect. A characteristic of asset returns is the accumulation of volatility, first observed by Mandelbrot (1963). Regardless of sign, large changes tend to be followed by large changes, and small changes tend to be followed by small changes (Fama, 1970). He also mentions periods of high and low volatility, stating that ``big price moves are followed by big price moves, but their magnitude is unpredictable." According to Deebom and Essi (2017) there are two types of volatility models, and they include symmetric models and asymmetric models. The main difference between these two classes is that symmetric models, such as ARCH and

GARCH, do not capture leverage effects in the time series, unlike asymmetric models (Deebom, Mazi, Chims, Richard &, George,2021).

GARCH models consider volatility clustering and the heteroskedasticity properties of variance and covariance, which are some of the characteristics of financial time series. GARCH models allow conditional fluctuations to depend on previous lags. In another development, The GARCH model transforms the AR process from an ARCH model to an ARMA process by adding an MA process (Wobo &Deebom, 2022). The standard GARCH model assumes that positive and negative error terms have symmetrical effects on volatility. In other words, both good and bad news have the same impact on the volatility model. This assumption is often violated in stock returns, as volatility increases more after bad news than after good news. To avoid marginal model misspecification, we consider all possible types of GARCH models, including asymmetric GARCH specifications such as standard GARCH(s-GARCH) and exponential GARCH (EGARCH). We also consider the leverage effects that exist in the series. Conditional densities used for error distributions such as integrated model GARCH (IGARCH), asymmetric power ARCH (apARCH), and Grosten-Jagannathan-Rankl GARCH (GJR-GARCH). Normal, (Normal) Student-t (std), Skew-student (ghst), skew-normal (snorm)(Deebom, *et al*, 2021). The motivation for this paper arises, first, from the fact that all his GARCH models are originally based on the assumption that financial time series follow a normal distribution (Gaussian distribution). However, significant evidence suggests that financial time series are rarely Gaussian but are usually leptokurtic and exhibit distinctive behavior. Theoretically, the GARCH model can accommodate fat-tailed distributions through its specification. However, GARCH models should use fat-tailed distributions, such as Student's t distribution, or other distributions that can produce more efficient results. Second, fat-tailed GARCH models are particularly important for accurately predicting financial volatility, which is important for portfolio risk management, such as value-at-risk and/or conditional tail risk - expected value measurements. The main drawback is that some distributions, even the widely used Student t distribution, lack aggregation stability. If you don't know the true distribution, you need to know which fat-tailed distribution to use for your GARCH model. This is important for portfolio applications and risk management and may further benefit portfolio management and other enterprise risk management issues, where accurate risk measurement is a major concern.

3. Methodology

Symmetric GARCH Models

The GARCH model is an extension of the ARCH model that was developed by Engle and Bollerslev (1986) which was applied to overcome the heterogeneity arising from high data volatility. The conditional variance is represented as a linear function of a long term mean of the variance, its own lags, and the previous realized variance. The simplest model specification is the GARCH (1,1) model:

Mean equation $r_t = \mu + \varepsilon_t$

Variance equation $\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

Where $\omega > 0, \alpha_1 > 0$ and

r_t = return of the asset at time t

$\mu =$ average return

$\varepsilon_t =$ residual returns, defined as

$\varepsilon_t = \sigma_t z_t$

Where z_t are standardized returns (i.e. realization of an iid random variable with zero mean and variance 1), and σ_t^2 stands for the conditional variance. For GARCH (1,1), the constraints $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ are needed to ensure that σ_t^2 is strictly positive (Poon,2005). The conditional variance equation models the time varying nature of volatility of the residuals generated from the mean equation. The definition of the GARCH model starts from the log-return series,r_t of an asset price P_t at time t given as,

$$r_t = \log(P_t) - \log(P_{t-1})$$

where P_{t-1} is the price at the previous time, $t-1$

Let the conditional mean of r_t given C_{t-1} be $E(r_t|C_{t-1}) = \mu_t$

and conditional variance of r_t given C_{t-1} be $Var(r_t|C_{t-1}) = E[(r_t - \mu_t)^2|C_{t-1}] = \sigma_t^2$,

where C_{t-1} is the information set available at the time $t-1$. The time series r_t is represented as the sum of a predictable and unpredictable part as

$$r_t = E(r_t|C_{t-1}) + \varepsilon_t$$

ε_t is conditionally heteroscedastic once $\varepsilon_t = z_t\sigma_t$, z_t follows a particular distribution, either Gaussian, Student t, and Generalized Error distributions or skewed version of these distributions and σ_t is the square root of the conditional volatility series. Therefore, Engle (1982) proposed modelling the residuals ε_t with the ARCH(q) model

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 = \omega + \alpha(L)\varepsilon_t^2$$

where the parameters $\omega > o,\ \alpha_i \geq 0$ for $i = 1, \dots, p$

$$\alpha(L) = \alpha L = \sum_{i=1}^{P} \alpha_i L^i$$

is the polynomial of order p defined for the ARCH parameter and $\alpha_1 + \beta_1 < 1$ to ensure covariance stationary conditional variance.

Bollerslev(1986) generalized Engle's model by including lags of unconditional variance in the model given as

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-j}^2$$

$$= \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \qquad (1)$$

where $\beta_i \geq 0$ for

$$j = 1, \dots, q, \beta(L) = \beta L = \sum_{j=1}^{q} \beta_j L^j$$

is the polynomial of order q defined for the GARCH parameters remain as defined above. Equation (11) can be represented as an autoregressive moving average (ARMA(p,q)) process as,

$$\varepsilon_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + V_t - \sum_{j=1}^{q} \beta_j V_{t-j} + (\sum_{j=1}^{q} \beta_i \sigma_{t-j}^2) \qquad (2)$$

where in compact form as,

$$\Phi(L)(1-L)\varepsilon_t^2 = \omega + (1 - \alpha(L))V_t$$

or,

$$\sigma_t^2 = \omega + (1 - \beta(L) - \Phi(L)(1 - L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \quad (3)$$

where

$$\Phi(L) = 1 - \sum_{i=1}^{p} \emptyset_i L^i$$

and L is the backward shift operator. From the model, there is second-order stationarity if the roots $\alpha(L) + \beta(L) = 1$ lie outside the unit circle. Since the estimate $\alpha(L) + \beta(L)$ is always very close to unity, this motivated the development of the integrated GARCH (IGARCH(p,q)) model of Engle and Bollerslev (1986) which is

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

This specification is often interpreted in a financial context, where an agent or trader predicts this period's variance by forming a weighted average of a long-term average (the constant), the forecast variance from last period (the GARCH term), and information about volatility observed in the previous period (the ARCH term). If the asset return was unexpectedly large in either the upward or the downward direction, then the trader will increase the estimate of the variance for the next period, while the GARCH-term generates persistence of volatility.
The basics GARCH(1,1) model can be written as an ARMA(1,1) model in terms of squared residuals.

The symmetric GARCH(p,q) model:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-j}^2$$

$$\text{for} \begin{cases} j = 1,2, \dots, q; \\ i = 1,2, \dots, p; \\ \omega_j > 0; \\ \beta_j \geq 0; \\ \beta_j < 1; \\ \alpha > 0 \end{cases}$$

Asymmetric GARCH Models

An interesting feature of asset prices is that bad news appears to have a stronger impact on volatility than good news. For many stocks, there is a strong negative correlation between current returns and future volatility. The tendency for volatility to decrease as returns increase and for volatility to increase as returns decrease is often referred to as the leverage effect (Enders, 2004). The main drawback of the symmetric GARCH model is that the conditional variance cannot respond asymmetrically to increases or decreases in aversion. Such effects are thought to be important for the behavior of stock returns. In a linear GARCH (p,q) model, the conditional variance is a function of the previous conditional variance and the squared innovation. Therefore, the sign of returns cannot affect volatility (Knight and Satchell, 2002). Therefore, since the symmetric GARCH model described above cannot explain the leverage effect observed on stock returns, several models are introduced to deal with this phenomenon. These models are called asymmetric models. EGARCH, TGARCH, and APGARCH capture asymmetric phenomena.

The Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) Model

The first nonlinear asymmetric volatility model is the EGARCH(1,1) model, which captures asymmetric responses of the time-varying variance to shocks and, at the same time, ensures that the variance is always positive.

$$\log \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} - E\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) \right| + \gamma_1 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) + \beta_1 log\sigma_{t-1}^2$$

with the parameters as defined in the GARCH model in (3) except $\gamma_1 \neq 0$ to allow for the asymmetric effect. It is noted that $log\sigma_t^2$ is linear in $z_t = {}^{\varepsilon_t}/_{\sigma_t}$ with slope $\alpha_1 + \gamma_1$ whenever z_t is over the range $0 < z_t < \infty$ and $log\sigma_t^2$ is also linear on $-\infty < z_t < 0$ with the slope $\alpha_1 - \gamma_1$. The α_1 gives the magnitude effect while the second term γ_k measures the asymmetric effect as in the ARCH model. The asymmetric representations of the models allow for both positivity (good news) and negativity (bad news) of the innovations to determine the variance.

The initial motivation of Nelson (1991) was to propose a model that could capture the asymmetric with the following simple specification:

$$log(\sigma_t^2) = \omega + \alpha_1 L_n(\sigma_{t-1}^2) + \beta_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

Where γ is the asymmetric response parameter or leverage parameter. The sign of γ is expected to be positive in most empirical cases so that a negative shock increases future volatility or uncertainty while a positive shock eases the effect on future uncertainty.

The Component Generalized Autoregressive Conditional Heteroscedastic (CGARCH) Model

The Component GARCH (CGARCH) model developed by Engle and Lee decomposes the conditional variance into a permanent and transitory component. This allows the investigation of the long- and short-run movements of volatility affecting securities in finance research.

The CGARCH(1,1) specification is

$$r_t = \mu + \varepsilon_t \qquad \text{where } \varepsilon_t = \eta_t \sqrt{h_t}$$
$$h_t = q_t + s_t$$
$$s_t = (\alpha + \beta) s_{t-1} + \alpha(\varepsilon_{t-1}^2 - h_{t-1})$$
$$q_t = \omega + \rho q_{t-1} + \varphi(\varepsilon_{t-1}^2 - h_{t-1})$$

Where ε_t is the error at time t. η_t is an identically and independently distributed with zero mean and unit standard deviation. h_t is the conditional variance of r_t at time t which is composed of a transitory component stand a permanent component qt. α + β and ρ measure the autoregressive persistence of the transitory and permanent components, respectively.α and φ stand for the immediate impacts of volatility shocks $(\varepsilon_{t-1}^2 - h_{t-1})$ on the short- and long-run components, respectively. It is constrained (α + β) < ρ to distinguish between the two components.

The Threshold Generalized Autoregressive Conditional Heteroscedastic (TGARCH) Model

Another GARCH variant that is capable to distinguish between positive and negative effects or good and bad news effects on volatility or handle leverage effects is the threshold GARCH (or TGARCH) developed by Zakoian (1994). In the TGARCH (1,1) version of the model, the specification of the conditional Variance is

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \gamma d_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

where $d_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ bad news
$d_{t-1} = 0$ if $\varepsilon_{t-1} \geq 0$ good news

Again, the coefficient γ is known as the asymmetry or leverage parameter. When $\gamma = 0$, the model collapses to the standard GARCH forms. Otherwise, when the shock is positive (i.e., good news) the effect on volatility is $\alpha_1$but when the news is negative (i.e., bad news) the effect on volatility is $\alpha_1 + \gamma$. Hence, if γ is significant and positive, negative shocks have a larger effect on σ_t^2 than positive shocks (Carter, 2007).

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i\, \varepsilon_{t-1}^2 \sum_{i=1}^{q} (\beta_1 + \gamma d_{t-1})\sigma_{t-1}^2$$

where ω is a constant, α_i is the coefficient of the lagged conditional variance ε_{t-1}^2, β_1 is a measure of a positive shock (good news),γ is a measure of asymmetric impact or leverage term, and negative shock(bad news) impact is measured by $\beta_1 + \gamma$.

The Asymmetry Power Generalized Autoregressive Conditional Heteroscedastic (APGARCH) Model

Ding, Granger and Engle (1993) also introduced the Asymmetry Power GARCH (APGARCH) specification to deal with asymmetry. Unlike other GARCH models, in this model, the standard deviation is modelled rather than the variance as in most of the GARCH-family. In Power GARCH an optional parameter γ can be added to account for asymmetry (Floros, 2008). The model also offers one the opportunity to estimate the power parameter σ instead of imposing it on the model (Ocran and Biekets, 2007). The general asymmetric Power GARCH model specifies σ_t as of the following form:

$$\sigma_t^\delta = \omega + \beta_1\sigma_{t-1}^\delta + \alpha_1(|\varepsilon_{t-1}| - \gamma_1\varepsilon_{t-1})^\delta$$

where $\delta > 0$ and $-1 < \gamma_1 < 1$, α_1 and β_1 are the standard ARCH and GARCH parameters, $\gamma_1$is the leverage parameters and γ_1 is the leverage for the power term. when $\delta = 2$, the equation becomes a classic GARCH model that allows for leverage effects, and when $\delta = 1$, the condition standard deviation will be estimated. This power parameter δ is estimated along with other parameters in the model. The APARCH model converges to GARCH(1,1) when the power parameter is squared ($d = 2$) and the asymmetric parameter nullified ($\gamma_1 = 0$).

Glosten Jagannathan and Runkle(GJR-GARCH(p,q)) model

To capture the asymmetry property under the sense that shocks not have the exact same impact on volatility in between negative and positive shocks, using GJR GARCH model which was proposed by Glosten

$$r_t = \beta_0 + \beta_1 r_{t-1} + \varepsilon_t$$
$$= \beta_0 + \beta_1 r_{t-1} + \sigma_t z_t$$
$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \theta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I_{t-1}$$

$$\text{for} \begin{cases} |\beta| > 0 \\ |\alpha| > 0 \\ \theta > 0 \\ \gamma > 0 \\ \alpha + \gamma > 0 \\ \varepsilon_t \sim N(0, \sigma^2) \\ (\varepsilon_{t-1} < 0) \end{cases}$$

where r_t is a market return at time t, $\gamma, \beta_0, \beta_1, \mu, \alpha, \theta$ are the parameters to be estimated and $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $I_{t-1} = 0$ if otherwise. Therefore, the model is said to allow good news$(\varepsilon_{t-1} > 0)$ and bad news$(\varepsilon_{t-1} < 0)$ to have differential effects on the conditional variance. $\varepsilon_t = \sigma_t z_t$ are the residual return and z_t is standardized residual that must be satisfy independently and identically distributed.

The basics GARCH (1,1) model can be written as an ARMA(1,1) model in terms of squared residuals. The symmetric GARCH(p,q) model:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-j}^2$$

The GARCH model above can easily be rewritten as

$$\emptyset(L)\varepsilon_t^2 = \alpha_0 + \beta(L)u_t$$

where $u_t = \varepsilon_t^2 - \sigma_t^2$

$$\emptyset(L) = 1 - \emptyset_1 L - \emptyset_2 L^2 - \cdots - \emptyset_p L^p$$

$$\beta(L) = 1 - \beta_1 L - \beta_2 L^2 - \cdots - b_q L^q$$

with $p = \max(d, p)\ and\ \emptyset_i = \alpha_i + \beta_i$

Generalised Autoregressive Conditional Heteroscedasicity (GARCH) methodology is used to model heteroscedacity in financial data, where ARMA process models the conditional mean and GARCH process describes the conditional variances. Combining the ARMA and GARCH model and described in the form of an apARCH(asymmetric power ARCH) model as in

$$x_t = \mu + \sum_{i=1}^{m} \alpha_i x_{t-i} + \sum_{j=1}^{n} \beta_j \epsilon_{t-j} + \epsilon_t$$

$$\epsilon_t = z_t \sigma_t$$

$$z_t \sim \mathfrak{D}_\vartheta(0,1)$$

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} a_i(|\in_{t-i}| - \gamma_i \epsilon_{t-i})^\delta \sum_{j=1}^{q} b_j \sigma_{t-j}^\delta$$

where x_t denotes the actual returns for asset, z_t is the standardized residuals with zero mean and unit variance and the parameter restrictions are $\omega, a_i, b_j \geq 0$, with mean μ, autoregressive coefficients α_i,moving coefficientsβ_i. $\mathfrak{D}_\vartheta(0,1)$ is the probability density function of the innovations or residuals with zero mean and unit variance. ϵ_t(ordinary residuals of the ARMA process) innovations. Alternatively ϑ are additional distribution parameters that are used to describe the skewness ξ with shape v of the parameter, γ_i shows the leverage effect (that is a positive $\gamma_i$means a negative shock has a stronger impact than a positive shock on the price volatility),b_j is the GARCH parameter, and δ is the exponent of conditional variance.

Distribution of Marginals

The realisation of the standardised residuals from the GARCH(p,q) model for the variables should follows any of the following marginal distributions ,$f_X(x)$ is the probability density function(pdf) of the marginals.

Generalized t Distribution

$$f_X(x) = \omega \left\{2\sigma\psi^{1/\omega} B\left(\frac{1}{\omega},\psi\right)[1+\frac{|K|^\omega}{\psi}]^{\psi+(\frac{1}{\omega})}\right\}^{-1}$$

for $\sigma,\psi,\omega \in \mathbb{R}^+, \mu, x\epsilon\mathbb{R}, k = {(x-\mu)}/{\sigma},$ and $B(.,.)$ is the beta function given by

$$\text{B}\,(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt, \text{ for } \alpha,\beta > o.$$

Skew Normal Distribution

$$f_X(x) = 2\emptyset(z)\Phi(\beta z); x,\beta,\mu \in \mathbb{R}, \sigma\mathbb{R}^+,$$

where $z = x - {\mu}/{\sigma}$, $\emptyset(\cdot)$ is the probability density function (pdf) of the standard normal distribution given by $f_Y(y) = ({1}/{\sqrt{2\Pi}})e\{^{-x^2}/_2\}$, $y \in (\infty,\infty)$ and $\Phi(\cdot)$ is its cumulative distribution function(cdf) given by $F_Y(y) = \left[\text{erf}\left(x/\sqrt{2}\right)+1\right], y \in (-\infty,\infty)$.

Leverage effect

Black (1976) first noted that changes in stock returns often display a tendency to be negatively correlated with changes in returns volatility, i.e., volatility tends to rise in response to bad news and to fall in response to good news. This phenomenon is termed the leverage effect and can only be partially interpreted by fixed costs such as financial and operating leverage (see Black, 1976; Christie, 1982).

Log-returns

The asset prices are transformed into log return series, $R_t$given by,

$$R_t = log_e\left(1+\frac{P_t - P_{t-1}}{P_{t-1}}\right) = log_e\left(\frac{P_t}{P_{t-1}}\right) = log_e P_t - log_e P_{t-1}$$

Where P_t is the current price of asset series of interest and P_{t-1} is the previous price of the asset. R_t is the log-returns series. To investigate volatility persistence, we take absolute and squared values of the return series as $|R_t|$and R_t^2 respectively.

Testing for Heteroscedasticity/ARCH effect

Financial time series are not immediately suitable for copula modelling because they are serially correlated (Patton, 2012). We need first to eliminate autocorrelation and seasonality. This can be achieve by testing for seasonality and volatility using AR or SARIMA model for seasonality and GARCH model for volatility. Testing for Heteroscedasticity is synonymous to testing ARCH effect. Thus, we are to test if the returns $r_t = c_0 + \varepsilon_t$ or $r_t = \varepsilon_t$ have constant variance.

Figure 1. Time Plot of Each Asset

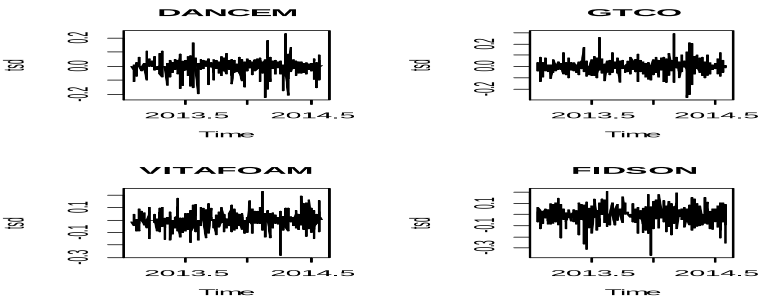

Table 1 Descriptive Statistics

		DANCEM	GTCO	NESTLE	VITAFOAM	FIDSON
N	Statistic	552	552	552	552	552
Range	Statistic	243	41.34	1045	23.07	14.69
Minimum	Statistic	117	13.37	570	1.43	0.81
Maximum	Statistic	360	54.71	1615	24.5	15.5
Sum	Statistic	113415.6	15546.14	630541.3	3983.19	2325.79
Jarque-Bera	Statistic	$\mathbf{1.061e^{-05}}$	8.882e-16	9.101e-09	2.2e-16	2.2e-16
Mean	Statistic	205.463	28.1633	1142.285	7.2159	4.2134
	Std. Error	1.99559	0.29531	11.24293	0.29865	0.11911
Std. Deviation	Statistic	46.88575	6.93822	264.149	7.01667	2.79835
Variance	Statistic	2198.273	48.139	69774.7	49.234	7.831
Skewness	Statistic	0.484	0.814	-0.092	1.384	1.613
	Std. Error	0.104	0.104	0.104	0.104	0.104
Kurtosis	Statistic	-0.243	0.627	-1.256	0.216	2.626
	Std. Error	0.208	0.208	0.208	0.208	0.208

Table 2 Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	Df	Sig.	Statistic	df	Sig.
DANCEM	.095	552	.000	.966	552	.000
GTCO	.109	552	.000	.952	552	.000
NESTLE	.124	552	.000	.947	552	.000
VITAFOAM	.309	552	.000	.684	552	.000

FIDSON	.160	552	.000	.835	552	.000
a. Lilliefors Significance Correction						

Table 3 : Estimated Coefficients of the EGARCH (1,1) model for DANCEM

	Estimate	Std. error	t-value	P- value
α_1	0.105278	0.086400	1.2185	0.223036
β_1	0.897593	0.064064	14.0109	0.000000
γ_1	0.373038	0.115700	3.2242	0.001263
Log-lik.	1087.602			
AIC	-3.9151			
BIC	-3.8446			

Estimated Coefficients of the EGARCH (1,1) model for GTCO

	Estimate	Std. error	t-value	P- value
α_1	-0.12891	0.057809	-2.22999	0.025748
β_1	0.84581	0.0681129	13.83647	0.000000
γ_1	0.37792	0.097788	3.86470	0.000111
Log-lik.	915.4166			
AIC	-3.2901			
BIC	-3.2197			

Estimated Coefficients of the EGARCH (1,1) model for NESTLE

	Estimate	Std. error	t-value	P- value
α_1	-0.238320	0.053312	-4.4703	0.000008
β_1	0.999990	0.000003	287725.037	0.000000
γ_1	0.028419	0.005662	5.0189	0.000001
Log-lik.	1170.909			
AIC	-4.2175			
BIC	-4.1470			

Estimated Coefficients of the GARCH (1,1) model for VITAFOAM

	Estimate	Std. error	t-value	P- value
α_1	-0.131912	0.116915	-1.12827	0.259206
β_1	0.487575	0.203137	2.40022	0.016385
γ_1	0.515104	0.150921	3.41307	0.000642
Log-lik.	887.247			
AIC	-3.1878			
BIC	-3.1174			

Estimated Coefficients of the GARCH (1,1) model for FIDSON

	Estimate	Std. error	t-value	P- value
α_1	-0.218581	0.052171	-4.1897	0.000028
β_1	1.000000	0.000028	36304.9430	0.000000
γ_1	0.131465	0.028682	4.5836	0.000005
Log-lik.	788.7876			
AIC	-2.8304			
BIC	-2.7600			

Table 3: GARCH MODELS FOR DANCEM

	Generalized t-distribution		Skew Normal distribution		Skew Student t-distribution		Normal Inverse Gaussian	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Standard GARCH	-3.8418	-3.7871	-3.5078	-3.4530	-3.8545	-3.7919	-3.9214	-3.8588
	LL=1085.427		LL=973.4008		LL=1069.906		LL=1088.35	
Exponential GARCH	-3.8527	-3.7901	-3.5114	-3.4488	-3.8543	-3.7839	-3.9151	-3.8446
	LL=1069.41		LL=975.3988		LL=1070.857		LL=1087.602	
CGARCH	-3.8260	-3.7556	-3.4974	-3.4270	-3.8169	-3.7387	-3.9226	-3.8444
	LL=1063.064		LL=972.5408		LL=1061.558		LL=1090.688	
GJR GARCH	-3.8393	-37767	-3.5112	-3.4486	-3.8509	-3.7805	**-4.3191**	**-4.2487**
	LL=1065.727		LL=975.3327		LL=1069.922		**LL=1198.909**	
Fractional Int.GARCH	-3.8863	-3.8080	-3.5095	-3.4312	-3.477	-3.4173	-3.8939	-3.8078
	LL=1080.67		LL=976.8563		LL=969.8738		LL=1083.775	

GARCH MODELS FOR GTCO

	Generalized t-distribution		Skew Normal distribution		Skew Student t-distribution		Normal Inverse Gaussian	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Standard GARCH	-3.2748	-3.2201	-3.1724	-3.1176	-3.2760	-3.2134	-3.2781	-3.2155
	LL= 909.22		LL= 880.98		LL= 910.5439		LL= 911.1156	
Exponential GARCH	-3.2873	-3.2247	-3.1756	-3.1130	-3.2814	-3.2110	**-3.2901**	**-3.2197**
	LL= 913.6466		LL= 882.8871		LL= 913.0265		**LL= 915.4166**	
CGARCH	-3.2719	-3.2015	-3.1638	-3.0934	-3.2617	-3.1834	-3.2719	-3.1936
	LL= 910.4096		LL= 880.6398		LL= 908.5936		LL= 909.6231	
GJR	-3.2852	-3.2226	-3.1762	-3.1136	-3.2825	-3.2121	-3.2869	-3.2164
	LL= 913.0845		LL= 883.0455		LL= 913.3362		LL= 914.5302	
Fractional Int. GARCH	-3.2784	-3.2002	-3.1739	-3.0956	-3.2757	-3.1896	-3.2829	-3.1968
	LL= 913.207		LL= 884.4026		LL=913.4423		LL= 915.4166	

GARCH MODELS FOR VITAFOAM

	Generalized t-distribution		Skew Normal distribution		Skew Student t-distribution		Normal Inverse Gaussian	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Standard GARCH	-3.1464	-3.0916	-2.9689	-2.9142	-3.1446	-3.0820	-3.1787	-3.1161
	LL=873.8203		LL=824.9436		LL=874.3311		LL=883.7294	
Exponential GARCH	-3.1522	-3.0896	-2.9855	-2.9229	-3.1487	-3.0783	-3.1878	-3.1174
	LL=876.4309		LL=830.5179		LL=876.4709		LL=887.247	
CGARCH	-3.1295	-3.0591	-2.9679	-2.8975	-3.1259	-3.0477	-3.1601	-3.0818
	LL=871.1902		LL=826.6693		LL=871.1931		LL=880.597	
GJR	-3.1609	-3.0983	-2.9087	-2.9181	-3.1577	-3.0873	**-3.5783**	**-3.5079**
	LL=878.8405		LL=829.1816		LL=878.9451		**LL=994.8202**	
Fractional Int. GARCH	-3.1124	-3.1011	-2.9550	-2.8767	-2.7707	-2.6846	-3.1783	-3.0922
	LL=871.0459		LL=824.1008		LL=774.3232		LL=886.6104	

GARCH MODELS FOR NESTLE

	Generalized t-distribution		Skew Normal distribution		Skew Student t-distribution		Normal Inverse Gaussian	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Standard GARCH	-3.8149	-3.7601	-3.3718	-3.3170	-3.8144	-3.7518	-4.0224	-3.9598
	LL=1057.993		LL = 935.933		LL= 1058.861		LL= 1116.73	
Exponential GARCH	-3.9616	-3.8990	-3.4046	-3.3420	-4.1024	-4.0320	**-4.2175**	**-4.1470**
	LL=1099.408		LL=945.9568		LL= 1139.222		**LL=1170.909**	
CGARCH	-3.3024	-3.3624	-3.3930	-3.3225	-3.8763	-3.7981	-4.1264	-4.0482
	LL=1104.238		LL=943.7617		LL=1077.927		LL= 1146.831	
GJR	-3.8124	-3.7498	-3.3905	-3.3279	-3.8090	-3.7386	-4.0328	-3.9624
	LL=1058.314		LL=1058.381		LL=1054.381		LL=1120.045	
Fractional Int. GARCH	-3.2293	-3.1511	-3.4013	-3.3231	-3.4013	-3.9616	-4.0122	-3.9212
	LL=899.6741		LL=947.0664		LL=	946.0575	LL=1134.742	

GARCH MODELS FOR FIDSON

	Generalized t-distribution		Skew Normal distribution		Skew Student t-distribution		Normal Inverse Gaussian	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Standard GARCH	-2.7437	-2.6889	-2.6323	-2.5775	-2.7458	-2.6832	-2.7946	-2.6320
	LL=762.884		LL=732.1975		LL=764.4671		LL=777.926	
Exponential GARCH	-2.7766	-2.7140	-2.6352	-2.5726	-2.7653	-2.6948	**-2.8304**	**-2.7600**
	LL=772.9529		LL=733.9881		LL=770.829		**LL=788.7876**	
CGARCH	-2.7598	-2.6894	-2.6314	-2.5610	-2.7574	-2.6791	-2.8042	-2.7260
	LL=769.3313		LL=733.9636		LL=769.654		LL=782.5636	
GJR	-2.7442	-2.6816	-2.6171	-2.5545	-2.7482	-2.6778	-2.8066	-2.7362
	LL= 764.0337		LL=729.0075		LL=766.128		LL=782.224	
Fractional Int. GARCH	-2.8133	-2.7401	-2.6542	-2.5759	-2.7608	-2.6747	-2.8829	-2.7968L
	LL=786.433		LL=741.2306		LL=771.5911		LL=805.2255	

Table 4: Marginal distribution of Dangote Cement

Distribution	AIC	BIC
Generalized t	-3.8863	-3.8080
Skew Normal	-3.5114	-3.4530
Student t distribution	-3.8545	-3.7919
Normal Inverse Gaussian	**-4.3191**	**-4.2487**

Marginal distribution of Guarantee Trust Company

Distribution	AIC	BIC
Generalized t	-3.2873	-3.2247
Skew Normal	-3.1756	-3.1176
Student t distribution	-3.2825	-3.2134
Normal Inverse Gaussian	**-3.2901**	-3.2197

Marginal distribution of Vitafoam

Distribution	AIC	BIC
Generalized t	-3.1609	-3.0983
Skew Normal	-2.9855	-2.9229
Student t distribution	-3.1577	3.0873
Normal Inverse Gaussian	**-3.5783**	**-3.5079**

Marginal distribution of Nestle

Distribution	AIC	BIC
Generalized t	-3.9616	-3.8990
Skew Normal	-3.4046	-3.3420
Student t distribution	-4.1024	-4.0320
Normal Inverse Gaussian	**-4.2175**	**-4.1470**

Marginal distribution of Fidson

Distribution	AIC	BIC
Generalized t	-2.8133	-2.7401
Skew Normal	-2.6542	-2.5775
Student t distribution	-2.7653	-2.6943
Normal Inverse Gaussian	**-2.8829**	**-2.7968**

RESULTS AND DISCUSSION

This section describes the results obtained by applying this methodology to our data. Some descriptive statistics and representation of variables are presented using weekly data of his five companies listed on the Nigerian Stock Exchange from February 2013 to February 2023. The companies are Dangote Cement (DANCEM), Guarantee Trust Holding Company (GTCO), Nestlé, Vitaform and Fidson. The plot clearly shows that the variable is not stationary. From the time graph of the data, it is easy to see that the data has high volatility. This supports his decision to use GARCH for modelling. The return series is appropriately biased, as indicated by the positive bias estimate. Based on skewness and kurtosis estimates, the return series does not follow a normal distribution. The results of this study are comparable to those of Deebom et al. (2021) “Comparative modelling of price fluctuations in the Nigerian crude oil market using symmetric and asymmetric GARCH models”. In Deebom et al. (2021) it was found that the estimates of skewness and kurtosis, and hence the return series, do not follow a normal distribution. Also, performing a Halke-Berra test on each of the log returns of the index using the null hypothesis that the data in each series comes from a normal distribution with unknown mean and unknown variance yields the null hypothesis was dismissed in his 5% S.I. Strictly reject normality for each row returned. Nestle's standard deviation of 264.149005 and Dangote Cement's standard deviation of 46.885487 in Table 1 confirm the high slope of the graph in Figure 1. Table 1 clearly shows that all assets are not normally distributed, regardless of the data period. The sample return skews are mostly positive, except for the Nestlé index, which has negative values. From the values obtained for kurtosis, all series in the sample appear to be clearly strong. This indicates non-normality and suggests that leptokurtosis arises from an accumulation pattern of market volatility, with periods of high (low) and periods of high volatility. (Low) volatility is followed numerically and graphically by period. This clearly suggests that ARMA and GARCH model specifications should be applied to deal with this non-normality condition. This is in line with Deebom and Essi (2017) who opined that the abnormality pattern of each asset is mainly due to the environment of the Nigerian financial market during the domestic and global financial crisis. Around 2016, Nigeria experienced a domestic financial crisis, a 20-year recession caused by economic collapse. This led to a sharp increase in bankruptcies, liquidations, and financial failures of households and businesses. The second volatility cluster was caused by the global financial crisis, which began with COVID-19 lockdowns and was followed by a recession.

To invest more accurately and strategically, you need to analyse the dependency structure of these assets, as descriptive statistics alone are not sufficient to analyse the relationships in multivariate data series. Common model selection criteria, Akaike Information Criterion (AIC) and Swartz-Bayesian Information Criterion (BIC), are used to select the best recommended model for a price and return series. The selection is based on the model with the lowest information criterion. From Table 4, we can see that the marginal values that best fit the residuals of his GARCH model for Dancem, GTCO, Vitafoam, Nestle, and Fidson were normal inverse Gaussian distributions, respectively. Non-normality and asymmetry are important for financial returns.

Conclusion

It can be concluded that time series descriptive statistics alone are not sufficient for the analysis of multivariate data series, as the dependence structure of assets needs to be analyzed. Therefore, further estimation is required to determine whether volatility, normal inverse Gaussian distribution, non-normality, and asymmetry are important for financial returns. Common model selection criteria, the Akaike Information Criterion (AIC) and the Swartz-Bayesian Information Criterion (BIC), can be used to determine which recommended model best fits the price and return series you choose. The results show that the best marginal values of the residuals of the GARCH model when estimating Dancem, GTCO, Vitafoam, Nestle, and Fidson were normal inverse Gaussian distributions, respectively.

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